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A More Realistic Solution to Refugee Housing Using the Isoperimetric Honeycomb Conjecture

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Over 6.6 million people live in refugee camps around the world. High-velocity wind storms frequently rip short-term structures in these camps apart, destroying what little refugees have left. Through the application of the Isoperimetric Honeycomb Conjecture and the Laws of Phi, we engineered a new, translationally symmetric icosahedron. This design minimizes the use of material needed by maximizing the number of isoperimetric hexagons and minimizing the number of pentagons while still creating a dome-like structure. After aerodynamic testing, we found the wind resistance of our structure to be 174.67 mph. Additionally, we engineered inexpensive, waterproof connectors to securely hold the structure together. These connectors were designed by developing a novel mathematical method involving calculating each dihedral angle using multivariable calculus and Euclidean geometry. We used three-dimensional vectors to calculate each cartesian point, then using the cross product found the normal vectors of each vertex. The point of intersections of these normal vectors were then inserted into an inverse cosine function giving us the dihedral angles needed for the connector design. This mathematical method was proven through the construction of our life-size model. The final 5-6 person structure is waterproof and fire-resistant, lasts 20 years, folds to the size of 1 backpack weighing 28.9 lbs, and costs $58.11 to make, which is 400-500 dollars cheaper than anything comparable on the market today. Also, our house follows the laws of special right triangles and therefore can be as small as a 1-2 person house or as big as a temporary hospital or school without additional structural support. The efficient, secure design, combined with the wind resistance of the structure, can help millions of refugees and has applications to help others such as military personnel or victims of natural disasters.

Introduction

War, persecution, and civil unrest have forced over 6.6 million people to abandon their homes and livelihoods and take refuge in one of many refugee camps around the world (*Shelter*, 2022). As we looked at the many problems that the world is facing, our attention turned to the news and the many refugees we saw there.

Current solutions to this housing problem are too expensive or inefficient in terms of transportation and ability to protect from weather environments such as dust storms (*What Is a Refugee Camp? Definition and Statistics: USA for UNHCR*, 2020). Due to the concentration of refugee camps in the Middle East, our design criteria focused on addressing high velocity dust storms, which are a prevalent issue in that geographic area (*What Is a Refugee Camp? Definition and Statistics: USA for UNHCR*, 2020).

After 5 years of development, we engineered a more realistic solution to refugee housing that is highly wind resistant, inexpensive, easily transportable, and lightweight using the mathematical principles of the "Isoperimetric Honeycomb Conjecture" and the laws of Phi. A 5-6 person version costs only

$58.11, folds down to 2 backpacks, and is made of treated corrugated plastic from recycled products. Proven through wind tunnel testing, the structure can resist wind speeds of over 160 mph.

This project has entered its final stages of development and will be presented to a nonprofit agency, like USAID (United States Agency for International Development), that can distribute this temporary housing solution worldwide.

Criteria

Net:

- Determine the dihedral angles between each individual plane of our structure and its surrounding planes
- Determine the drag coefficient, exposed surface area, and footing anchorage requirements
- Build a full-size model and test in a natural environment
- Build a ⅓ scaled model and test in a professional wind tunnel facility
- Develop a business model and manufacturing process on how to fund the building and shipping of these homes

Connectors: Design and build connectors which will securely fasten the hexagons and create a waterproof seal

- Connect panels together
- Water tight seal
- Wind resistant
- Removable
- Universal fit to any angle

Design Development

Net Design

The engineering criteria for our refugee housing solution was a structure that weighed less than 50 pounds, was easily transportable, cost less than $100 per house, and could resist winds of over 70 mph.

Domes are the most wind resistant restructures. However, current dome designs, such as the rigel, sirius, or antare are too tall to withstand wind storms, too flat to live in, or use too much material due to the number of pentagons. Thus, the creation of a new dome using the Isoperimetric Honeycomb Conjecture (Hales, 1999) was necessary.

The conjecture stated that a net of isoperimetric hexagons bent into a three-dimensional shape would create the largest volume-to-surface-area ratio than any other shape. By following this conjecture, the design maximizes volume, but minimizes the amount of material used.

Additionally, according to the Laws of Phi (Meisner & Seni, 2014), when a pentagon is placed in a system of hexagons it creates the correct curvature needed to create a dome. Thus, the incorporation of pentagons in the design is required.

However, pentagons do not follow the Isoperimetric Honeycomb Conjecture and therefore material cost would increase as more pentagons are added.

Through the engineering process involving maximizing hexagons and minimizing pentagons a new translationally symmetric truncated icosahedron was created, resulting in a hexadome that used 75 hexagons and 4 pentagons that are connected on two sides only (figure 1). Thus, the dome folds down, in an accordion manner, to the size of 2 backpacks.

Our first attempt to minimize pentagons used only 1 pentagon and 48 hexagons. However, the model had many gaps and overlaps. We took a different approach to building the dome; instead of starting with a 3-dimensional model, we started with a 2-dimensional net. The angles of the hexagons did not meet up and several gaps formed. We did notice in this model a "flower" shape that repeated itself several times. We used this flower shape in our next three designs. In the third design, we put the flower on the top. This design created the most dome-like dimensions, however it still had pointed corners. Then, after researching the Laws of Phi, we placed pentagons in the design, resulting in a hexadome that used 75 hexagons and 4 pentagons.

The hexagons in the structure follow the laws of special right triangles and therefore can be scaled up to a 15-20 person size structure, which could be used as a temporary hospital or school, without needing additional support.

Furthermore, the design will be made out of treated corrugated plastic, a material made out of recycled water bottles and treated with water, fire, mold and UV resistance that is sustainable for up to twenty years and weighs less than one pound per sheet.

Dihedral Calculations

For the development of connectors to join the polygonal panels together once the design is unfolded, the exact dihedral angles of the translationally symmetric truncated icosahedron needed to be calculated.

In a truncated icosahedron, the angle between a pentagon and a hexagon is 142.62º and the angle between two hexagons with a Pentagon base is 138.2º. It can be deduced that the center hexagon of the refugee housing design must be parallel to the ground because of the translationally symmetric relationship of the four pentagons in the design.

The multivariable calculus principle of three-dimensional vectors (Helfert & Daubenschüz, 2012) was used in order to determine the angle of tilt of each polygon, which was done by drawing each row of hexagons as an individual line and then orienting the vectors in relation to each other and then using trigonometric functions to find the cartesian points (figure 2).

After calculating these points, the z-coordinate was found by using the 3D Pythagorean theorem. Additionally, the cartesian points were reoriented to the center hexagon using Euclidean Geometry.

Once all of the points of the translationally symmetric polyhedron were on a 3D x,y,z plane, vector groupings were identified. These groupings contained three points, V0, V1, V2, and V3 (figure 3).

Where V1, V2, and V3 all had the initial starting point V0. Each vector was an edge between two hexagons. Then, W1, W2, and W3 were found by subtracting the corresponding x,y,z coordinates.

The cross product of W1xW2 and W2xW1 was then taken, which gave the normal vectors (U1, U2) of the two hexagons which met at line V0-V2. To calculate the intersection of these two normal vectors the dot product of U1 and U2 was taken (figure 4).

Finally, the inverse cosine function to find the angle at which the two normal vectors crossed, which is the dihedral angle between the hexagons that intersect at line V0-V2 was taken. In order to calculate the angles of tilt, the dihedral was subtracted from 180^{o}.

Connector Design

The mechanism that connects the hexagonal and pentagonal panels of our structure has three main elements:

1. The connection to the panel
2. The connection to other connectors on the edge of the panel
3. The connection to other connectors on the vertices of the panel

The criteria for the connector was watertightness up to 60 PSI, the ability to conform to any dihedral angle on the structure, a cost less than $0.16 per connector, and a weight less than 0.138 lbs per connector.

Throughout the process, considerations had to be taken for simplicity, cost, transportability, and the ability of the connector to be easily fixed in the event of breakage as well as for not only if the model would work between two panels, but if it would work in a system of connectors as well and if the design was practical to be used by individuals in a refugee camp.

Through extensive research and refinement this connector prototype will meet all of the engineering criteria established: watertight, universal fit to any dihedral angle, lightweight, inexpensive, and durable.

Testing

The data was converted to its estimated full scale velocity using the principle of Froude's Scaling Laws, which states the following:

$$Froude's\ Scaling\ Law:\ \mu_{Scale} = \lambda^{\frac{1}{2}}{}_{Scale} \cdot \mu_{Full}$$
$$and\ Fr_{Scale} = Fr_{Full}$$

$$Where\ Fr = \frac{\mu^2}{gD}$$

$$Thus: \frac{\mu^2}{gD} = \frac{\mu^2}{gD}$$

$$therefore \frac{\lambda^{\frac{1}{2}}{}_{Scale}((\lambda_{Full} \cdot v)^2)}{gD_{Scale}} = \frac{\mu^2}{(g(D_{Scale}\lambda_{Full}))} \lambda^{\frac{1}{2}}{}_{Scale}((\lambda_{Full} \cdot v)^2)$$
$$\cdot (g(D_{Scale}\lambda_{Full}))$$
$$= gD_{Scale}V^2$$

Thus to find the final full model velocity using the scaled data:

$$V_{Full} = \sqrt{\frac{\left(\lambda^{\frac{1}{2}}{}_{Scale}\right)\left((\lambda_{Full} \cdot v)^2\right)(g(D_{Scale}\lambda_{Full}))}{gD_{Scale}}}.$$

Scaled data values result in margin of errors which are dependent upon the degree of the model. The smallest scale design which is 1/20 the size shows results of a high predicted velocity and carries a negative exponential curve resulting in the expected line of best fit. Thus, the predicted value using the resulting estimated asymptote is 74.46 m/s or 166.56 mph.

Constraints

- The door to the wind tunnel will only fit a ⅓ size model
- The weights must be added from the bottom of the structure, so once the structure becomes too heavy for us to slide over the door in the wind tunnel, we cannot test anymore weights
- We only had 75 minutes to test in the wind tunnel

Aerodynamic Results

Full Size Model	Weight	Average Wind Speed at Bending (mph)	Average Wind speed when slid (m/s)	Average Wind speed when slid (mph)
2-3 People	300 lbs	134.25 mph	67.20 m/s	150.32 mph
5-6 People	420 lbs	134.25 mph	78.09 m/s	174.67 mph

Conclusion

Additional Applications

Besides refugee houses, our design has other possible applications around the world.

The structure can be incorporated into the emergency kits of the general population in case of catastrophe. The dome can be distributed to victims of hurricanes, earthquakes, or other natural disasters through humanitarian organizations. The geodesic dome can also be given to soldiers in the army, replacing their bulky tents with an easily transported accordion net.

Because the structure uses special right triangles, it can be blown up to any proportions; thus large scale domes can be used for temporary hospitals, schools, and service centers, while small domes can be created to house individual families.

Implications

The engineered connectors provide an ultra watertight seal for all of the predetermined angles, which were found by using an unprecedented mathematical method utilizing principles from multivariable calculus of three dimensional vectors, trigonometric functions, cross product and an inverse cosine function using the dot product.

Our final design was a dome-like structure that resists winds of over 160 mph, costs $58.11, weighs 28.9 lbs and can be folded down to the size of 2 backpacks.

The final stage of development includes testing our design in simulated extreme weather environments including fluctuating temperatures, hurricane force rain, high altitude wind velocities, and sandstorm particulate testing. Additionally, a study in mass scale manufacturing and distribution is needed in order to donate to a nonprofit like USAID or UNHCR.

References

Hales, T. (1999). Isoperimetric Honeycomb Conjecture. *Arxiv*, 9906042. http://arxiv.org/pdf/math/9906042.pdf

Helfert, S. F., & Daubenschüz, M. (2012). Full Vectorial Band Structure Computation of Photonic Crystals with Hexagonal Lattice and a Staircase Approximation of the Slanted Unit Cells. *OPTICAL AND QUANTUM ELECTRONICS*, *45*(4), 317–327. https://doi.org/10.1007/s11082-012-9637-1

Meisner, G., & Seni, L. (2014). Bucky Balls and Phi. *The Golden Ratio: Phi*, *1*(618). http://www.goldennumber.net/bucky-balls/

Shelter. (2022). Alternatives to Camps. | USA for UNHCR. https://www.unhcr.org/en-us/shelter.html

What Is a Refugee Camp? Definition and Statistics: USA for UNHCR. (2020). What Is a Refugee Camp? Definition and Statistics | USA for UNHCR. http://www.unrefugees.org/refugee-facts/camps/